

Today you will:

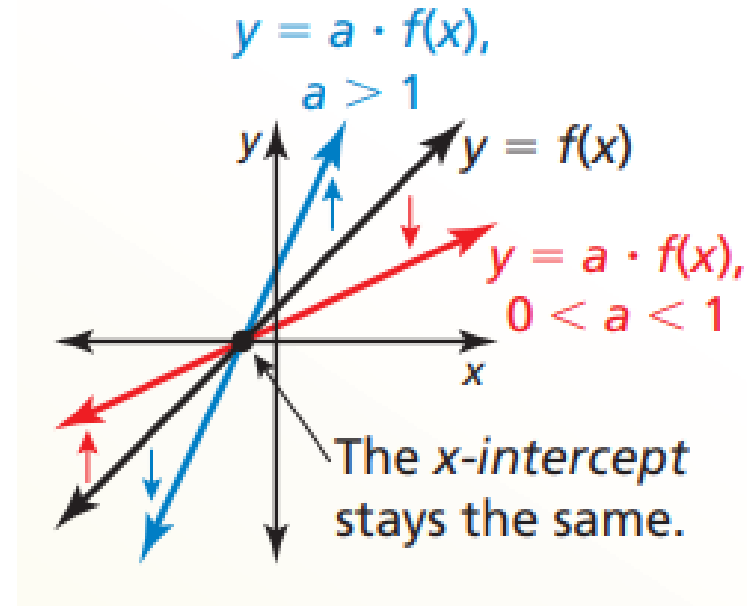
- Write functions representing stretches and shrinks
- Write functions representing combinations of transformations
- Practice using English to describe math processes and equations

Core vocabulary:

- Vertical Stretch (of a graph of a function) – move the points of the function away from x-axis by a factor
- Vertical Shrink (of a graph of a function) – move points of the function toward x-axis by a factor
- **Horizontal Stretch – move the points of the function away from the y-axis by a factor**
- **Horizontal Shrink – move the points of the function toward y-axis by a factor**

Vertical Stretches and Shrinks

- The graph of $y = a \cdot f(x)$ is a vertical
 - Stretch by a factor of a when $a > 1$
 - Shrink by a factor of a when $0 < a < 1$



Example of a Vertical Stretch:

$f(x) = 2x^2$ is a vertical stretch of $f(x) = x^2$ by a factor of 2

Example of a Vertical Shrink:

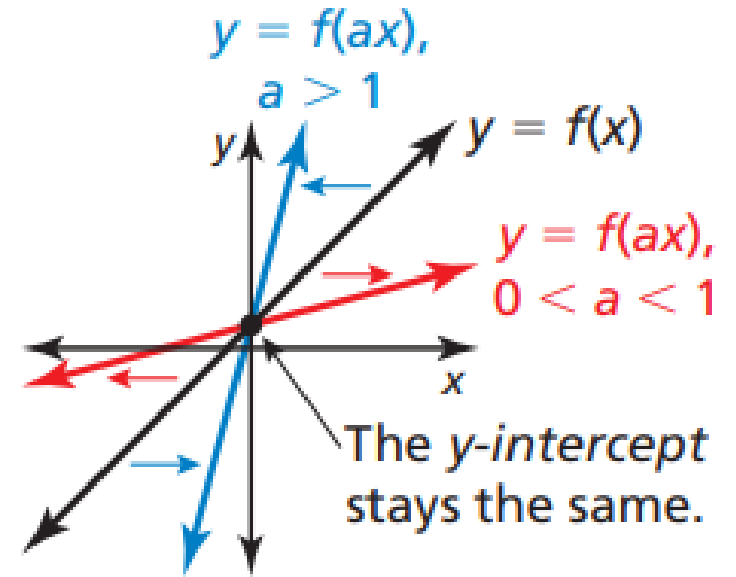
$f(x) = \frac{1}{2}x^2$ is a vertical shrink of $f(x) = x^2$ by a factor of $\frac{1}{2}$

*** Notice with a vertical stretch/shrink we are multiplying all the y's deforming the graph up/down.

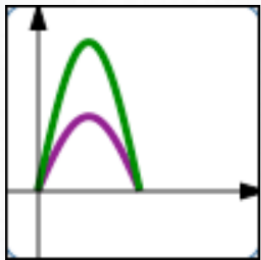
Horizontal Stretches and Shrinks

Horizontal stretches/shrinks are away from/toward the y-axis.

- The graph of $y = f(ax)$ is a horizontal
 - Stretch (away from y-axis) by a factor of $\frac{1}{a}$ when $0 < a < 1$
 - Shrink (toward the y-axis) by a factor of $\frac{1}{a}$ when $a > 1$



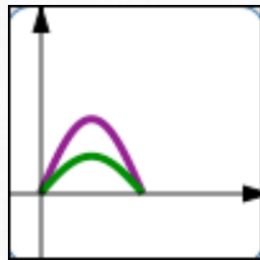
*** Notice with a horizontal stretch/shrink we are multiplying all the x's deforming the graph left/right.



$$y = f(x)$$

$$y = 2f(x)$$

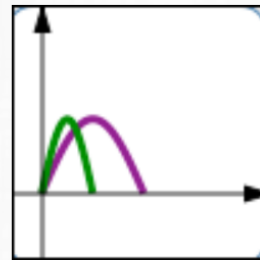
vertical stretch;
 y -values are doubled;
 points get farther away
 from x -axis



$$y = f(x)$$

$$y = \frac{f(x)}{2}$$

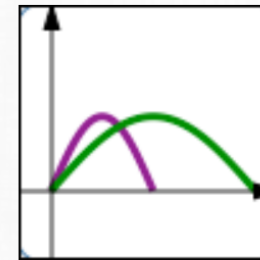
vertical shrink;
 y -values are halved;
 points get closer
 to x -axis



$$y = f(x)$$

$$y = f(2x)$$

horizontal shrink;
 x -values are halved;
 points get closer
 to y -axis



$$y = f(x)$$

$$y = f\left(\frac{x}{2}\right)$$

horizontal stretch;
 x -values are doubled;
 points get farther away
 from y -axis

vertical stretching/shrinking changes the y -values of points;
 transformations that affect the y -values are intuitive

horizontal stretching/shrinking changes the x -values of points;
 transformations that affect the x -values are counter-intuitive

Vertical/Horizontal Stretching/Shrinking usually changes the shape of a graph.

Let $f(x) = |x - 3| - 5$. Write (a) a function g whose graph is a horizontal shrink of the graph of f by a factor of $\frac{1}{3}$, and (b) a function h whose graph is a vertical stretch of the graph of f by a factor of 2.

SOLUTION

a. A horizontal shrink by a factor of $\frac{1}{3}$ multiplies each input value by 3.

$$\begin{aligned}g(x) &= f(3x) \\ &= |3x - 3| - 5\end{aligned}$$

Multiply the input by 3.

Replace x with $3x$ in $f(x)$.

► The transformed function is $g(x) = |3x - 3| - 5$.

b. A vertical stretch by a factor of 2 multiplies each output value by 2.

$$\begin{aligned}h(x) &= 2 \cdot f(x) \\ &= 2 \cdot (|x - 3| - 5) \\ &= 2|x - 3| - 10\end{aligned}$$

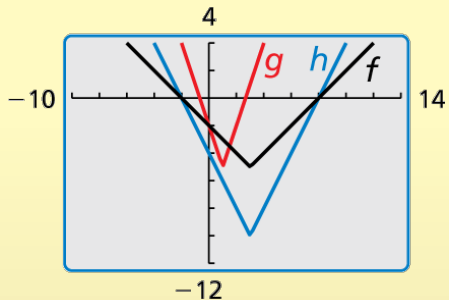
Multiply the output by 2.

Substitute $|x - 3| - 5$ for $f(x)$.

Distributive Property

► The transformed function is $h(x) = 2|x - 3| - 10$.

Check



Combinations of transformations:

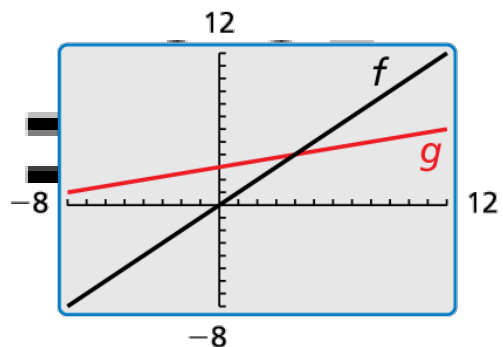
When told to apply a series (group) of transformations to a function,

apply them one at a time in the stated order.

Let the graph of g be a vertical shrink by a factor of 0.25 followed by a translation 3 units up of the graph of $f(x) = x$. Write a rule for g .

SOLUTION

Check



Step 1 First write a function h that represents the vertical shrink of f .

$$\begin{aligned}h(x) &= 0.25 \cdot f(x) \\ &= 0.25x\end{aligned}$$

Multiply the output by 0.25.

Substitute x for $f(x)$.

Step 2 Then write a function g that represents the translation of h .

$$\begin{aligned}g(x) &= h(x) + 3 \\ &= 0.25x + 3\end{aligned}$$

Add 3 to the output.

Substitute $0.25x$ for $h(x)$.

► The transformed function is $g(x) = 0.25x + 3$.



You design a computer game. Your revenue for x downloads is given by $f(x) = 2x$. Your profit is \$50 less than 90% of the revenue for x downloads. Describe how to transform the graph of f to model the profit. What is your profit for 100 downloads?

SOLUTION

- 1. Understand the Problem** You are given a function that represents your revenue and a verbal statement that represents your profit. You are asked to find the profit for 100 downloads.
- 2. Make a Plan** Write a function p that represents your profit. Then use this function to find the profit for 100 downloads.
- 3. Solve the Problem**

Vertical shrink by a factor of 0.9

$$p(x) = 0.9 \cdot f(x) - 50$$

Translation 50 units down

$$= 0.9 \cdot 2x - 50$$

Substitute $2x$ for $f(x)$.

$$= 1.8x - 50$$

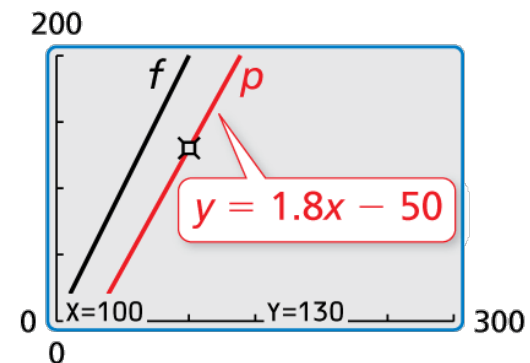
Simplify.

To find the profit for 100 downloads, evaluate p when $x = 100$.

$$p(100) = 1.8(100) - 50 = 130$$

► Your profit is \$130 for 100 downloads.

- 4. Look Back** The vertical shrink decreases the slope, and the translation shifts the graph 50 units down. So, the graph of p is below and not as steep as the graph of f .



Homework: pg 17 #17-32, 34

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